MATEMATIKA 11

ALGEBRA VA ANALIZ ASOSLARI
GEOMETRIYA
II QISM

Umumiy o‘rta ta’lim maktablarining 11-sinflari va o‘rta maxsus, kasb-hunar ta’limi muassasalari uchun darslik

O‘zbekiston Respublikasi Xalq ta’limi vazirligi tomonidan tasdiqlangan

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Darslikning “Algebra va analiz asoslari” bo‘limida ishlatilgan belgilar va ularning talqini:

△ – masalani yechish (isbotlash) boshlandi

▲ – masalani yechish (isbotlash) tugadi

(!) – nazorat ishlari va test (sinov) mashqlari

(?) – savol va topshiriqlar

■ – asosiy ma’lumot

* – murakkabroq mashqlar

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Algebra va analiz asoslari

II BOB. INTEGRAL VA UNING TATBIQLARI

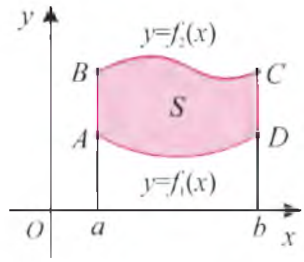
47–50 ANIQ INTEGRALNING TATBIQLARI

Yuzlarni integrallar yordamida hisoblash

Masala. Rasmdagi *ABCD* shakl yuzi *S* hisoblansin (7.a-rasm).

△ Ravshanki, bu shaklning *S* yuzi *aBCb* va *aADb* egri chiziqli trapetsiyalar yuzlarining ayirmasiga teng:

$$S=\int_a^b f_2(x)dx-\int_a^b f_1(x)dx=\int_a^b (f_2(x)-f_1(x))dx. \quad (1)$$



7.a-rasm.

Javob: $S=\int_a^b (f_2(x)-f_1(x))dx.$ ▲

(1) formula $f_2(x)\geq f_1(x)$ shartni qanoatlantiradigan uzluksiz funksiyalar uchun to‘g‘ridir.

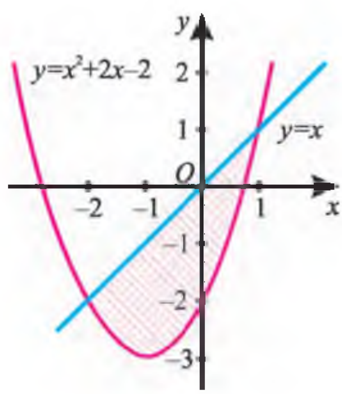
1-misol. $y=x$ to‘g‘ri chiziq va $y=x^2+2x-2$ parabola bilan chegaralangan shakl yuzini hisoblang.

△ 1) $y=x$ va $y=x^2+2x-2$ chiziqlarning kesishish nuqtalarini topamiz:

2) $x^2+2x-2=x$ tenglamadan $x_1=-2$, $x_2=1$.

Demak, chiziqlar (1; 1), (–2; –2) nuqtalarda kesishadi. Ravshanki, (–2; 1) oraliqda $y=x$ funksiya grafigi $y=x^2+2x-2$ funksiya grafigidan yuqorida yotadi (8-rasm).

U holda (1) formulada $a=-2$, $b=1$, $f_2(x)=x$, $f_1(x)=x^2+2x-2$ desak, izlanayotgan yuz (1) ga ko‘ra



8-rasm.

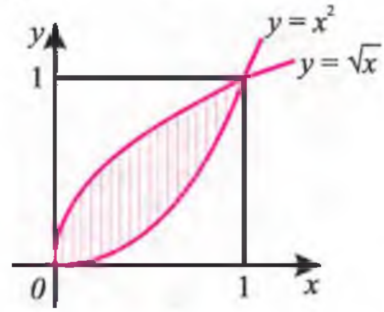
$$S=\int_{-2}^{1}\left(x-(x^2+2x-2)\right)dx=\int_{-2}^{1}(-x^2-x+2)dx=(-\frac{x^3}{3}-\frac{x^2}{2}+2x)\Big|_{-2}^{1}=\frac{7}{6}-(-\frac{10}{3})=4,5.$$

Javob: $S=4{,}5$ (kv.birlik). ▲

2-misol. $y=\sqrt{x}$ va $y=x^2$ chiziqlar bilan chegaralangan shakl yuzini hisoblang.

△ $x\in[0;1]$ kesmada $x^2\le\sqrt{x}$ (9-rasm).

(1) formulada $a=0$, $b=1$, $f_1(x)=x^2$, $f_2(x)=\sqrt{x}$ deymiz.



9-rasm.

U holda $S=\int_0^1(\sqrt{x}-x^2)dx=(\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{3}x^3)\Big|_0^1=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$(kv.birlik).

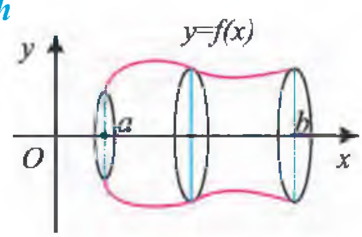
Javob: $S=\frac{1}{3}$ kv.birlik. ▲

Aylanish jismlarining hajmini hisoblash

Egri chiziqli trapetsiyani Ox o‘qi atrofida aylantirish natijasida hosil bo‘ladigan jismning hajmi

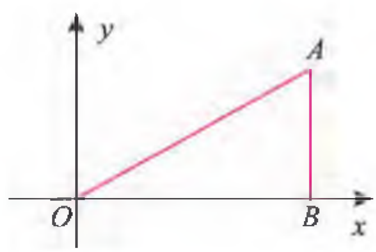
$$V=\pi\cdot\int_a^b f^2(x)dx \qquad (2)$$

formula bilan hisoblanishini isbotlash mumkin. Bu formuladan $f(x)$ ni tanlash hisobiga kesik konus, konus, silindr, shar, shar segmenti hajmlarini osonlikcha topsa bo‘ladi (10- rasm).



10-rasm.

Konusning hajmi. Bu holda $AB=R$, $OB=H$ deb olamiz (11-rasm). OA to‘g‘ri chiziq tenglamasi $y=\frac{R}{H}x$ ekanligi ravshan. U holda (2) formulaga muvofiq



11-rasm.

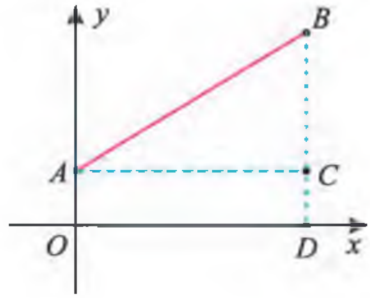
$$V_{konus}=\pi\cdot\int_0^H(\frac{R}{H}x)^2dx=\pi\cdot\frac{R^2}{3H^2}\cdot x^3\Big|_0^H=\pi\cdot\frac{R^2}{3H^2}\cdot(H^3-0)=\frac{1}{3}\pi R^2H.$$

Demak, $V_{konus}=\frac{1}{3}\pi R^2H.$

Kesik konusning hajmi. *AB* kesmani *Ox* o'qi atrofida aylantirishdan kesik konus hosil bo'ladi. $AO=r$, $BD=R$, $OD=H$ deylik (12- rasm).

AB to'g'ri chiziqning tenglamasi $y=\frac{R-r}{H}x+r$ ekani ravshan.

Demak, $a=0$, $b=H$, $f(x)=\frac{R-r}{H}x+r.$



12-rasm.

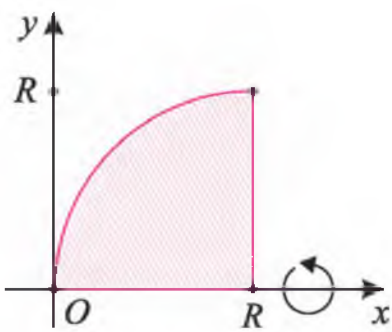
U holda (2) formulaga muvofiq

$$V_{k.konus}=\pi\cdot\int_0^H\left(\frac{R-r}{H}x+r\right)^2dx=\frac{\pi}{3}\cdot\frac{H}{R-r}\cdot\left(\frac{R-r}{H}x+r\right)^3\Bigg|_0^H=\frac{\pi}{3}\cdot\frac{H}{R-r}\cdot(R^3-r^3)=\frac{\pi}{3}\cdot H\cdot(R^2+Rr+r^2).$$

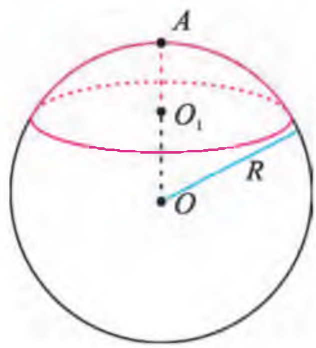
Shunday qilib, kesik konusning hajmi: $V=\frac{\pi}{3}\cdot(R^2+Rr+r^2)H$. Bundan $AO=r=0$ bo'lsa, konus hajmi formulasini olamiz.

Sharning hajmi. Radiusi *R*, markazi $(R;O)$ nuqtada bo'lgan doiraning chorak qismini *Ox* o'qi atrofida aylantirishdan (13-rasm) hosil qilinadigan shakl sharning yarmidir. Bizning holda mos aylana tenglamasi $(x-R)^2+y^2=R^2$ bo'ladi, bundan $y=\sqrt{2Rx-x^2}$, $x\in[0;R]$. (2) formulaga ko'ra

$$\frac{1}{2}V_{shar}=\pi\cdot\int_0^R(2Rx-x^2)dx=\pi\cdot\left(Rx^2-\frac{x^3}{3}\right)\Bigg|_0^R=\frac{2}{3}\pi R^3,\ \text{demak, } V_{shar}=\frac{4}{3}\pi R^3.$$



13-rasm.



14-rasm.

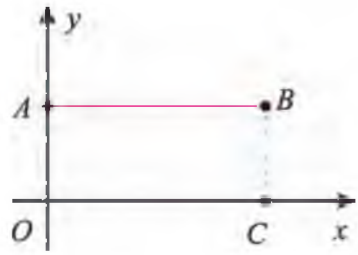
Shar segmentining hajmi. 14-rasmda $OA=R$, $O_1A=H$ (segmentning balandligi) bo‘lsin. Doira segmentini uning balandligi atrofida aylanishidan shar segmenti hosil bo‘ladi (14-rasm.) Shar segmentining hajmini hisoblash shar hajmini topish kabi bo‘ladi, bu holda integrallash $[0; H]$ kesma bo‘yicha bajariladi:

$$V_{segment}=\pi\cdot\int_0^H(2Rx-x^2)dx=\pi\cdot\left(Rx^2-\frac{x^3}{3}\right)\Big|_0^H=\pi\cdot\left(RH^2-\frac{1}{3}H^3\right).$$

Demak, $V_{segment}=\frac{1}{3}\pi\cdot H^2\cdot(3R-H)$.

Silindrning hajmi. *Ox* o‘qqa parallel *AB* kesmani *Ox* o‘qi atrofida aylantirishdan hosil bo‘ladigan shakl silindr bo‘ladi.

$AB=OC=H$, $OA=BC=R$ bo‘lsin (15-rasm). *AB* to‘g‘ri chiziq tenglamasi $y=R$ ekani ravshan, $x\in[0; H]$. U holda (2) formulaga ko‘ra,



15-rasm.

$$V_{silindr}=\pi\cdot\int_0^H R^2dx=\pi R^2x\Big|_0^H=\pi R^2\cdot(H-0)=\pi R^2H.$$ Demak, $V_{silindr}=\pi R^2\cdot H$.

Nega aylanish *Ox* o‘qi atrofida bo‘lishi kerak? Aylanish *Oy* atrofida bo‘lsa-chi? Bunday savolni qo‘yish tabiiy.

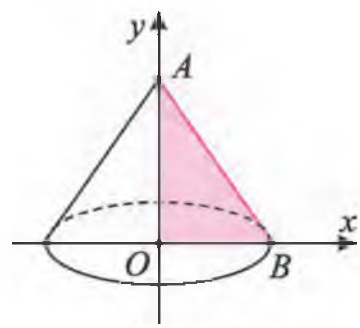
Yuqoridan uzluksiz $y=f(x)$ funksiya grafigi, pastdan *Ox* o‘qi, chap va o‘ngdan, mos ravishda, $x=a$ va $x=b$ vertikal chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning *Oy* o‘qi atrofida aylanishidan hosil bo‘ladigan jismning hajmi

$$V=2\pi\cdot\int_a^b xf(x)dx \qquad (3)$$

formula bilan hisoblanishini isbotlash mumkin.

1-misol. Konus hajmini toping (16-rasm).

△ $OA=H$, $OB=R$ deylik. *AB* to‘g‘ri chiziq tenglamasi $y=-\frac{H}{R}x+H$ ekani ravshan. U holda (3) formulada $a=0$, $b=R$, $f(x)=-\frac{H}{R}x+H$ desak, *AB* kesmaning *Oy* o‘qi atrofida aylanishidan hosil bo‘ladigan konus hajmi

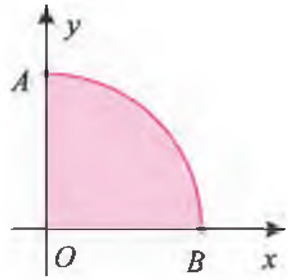


16-rasm.

$$V_{konus}=2\pi\cdot\int_0^R x\cdot(-\frac{H}{R}x+H)dx=2\pi\left[-\frac{H}{R}\int_0^R x^2dx+H\int_0^R xdx\right]=$$

$$=2\pi\cdot\left(-\frac{H}{R}\cdot\frac{x^3}{3}\right)\Bigg|_0^R+H\cdot\frac{x^2}{2}\Bigg|_0^R=-2\pi\cdot\frac{H}{R}\cdot\frac{R^3}{3}+2\pi\cdot H\cdot\frac{R^2}{2}=\pi R^2H\cdot(-\frac{2}{3}+1)=\frac{1}{3}\pi R^2H.$$

Demak, $V_{konus}=\frac{1}{3}\pi R^2H$. ▲



17-rasm.

2-misol. Radiusi R bo‘lgan shar hajmini toping.

△ $OA=OB=R$, O – aylana markazi deylik. Bu aylana tenglamasi, ravshanki, $x^2+y^2=R^2$, bundan $y=\sqrt{R^2-x^2}$, $0\le x\le R$. Bunga mos doiraning chorak qismini (17-rasm) Oy o‘qi atrofida aylantirishdan sharning yarmi hosil bo‘ladi. Avval shu yarimshar hajmini topamiz. (3) formulada $a=0$, $b=R$, $f(x)=\sqrt{R^2-x^2}$ deylik. U holda
$V=2\pi\cdot\int_0^R x\sqrt{R^2-x^2}dx.$

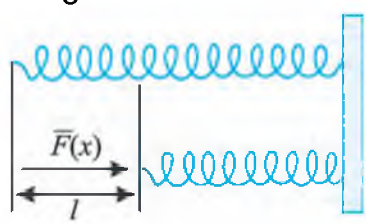
$R^2-x^2=u$ desak, $xdx=-\frac{du}{2}$,

$$\int x\sqrt{R^2-x^2}dx=-\frac{1}{2}\cdot\int\sqrt{u}du=-\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}+C=-\frac{1}{3}\cdot u^{\frac{3}{2}}+C=-\frac{1}{3}(R^2-x^2)^{\frac{3}{2}}+C.$$

Bu yerda $C=0$ deb olish mumkin.

Demak, $V=-\frac{2\pi}{3}(R^2-x^2)^{\frac{3}{2}}\Big|_0^R=\frac{2}{3}\pi R^3$ yoki $V_{shar}=\frac{4}{3}\pi R^3$. ▲

Kuchning bajargan ishini hisoblash. Vintsimon prujinaning bir uchi mustahkamlangan, ikkinchi uchiga esa $F=F(x)$ kuch ta’sir etib, prujinani siqadi, deylik (18-rasm). Guk qonuniga ko‘ra prujinaning siqilishi unga ta’sir etayotgan $F(x)$ kuchga proporsionaldir. Prujinani l birlikka siqish uchun $F(x)$ kuchning bajargan ishini toping.



18-rasm.

△ Ma’lumki, o‘zgaruvchi $F(x)$ kuchning $[a; b]$ oraliqdagi bajargan ishi

$$A=\int_a^b F(x)dx \qquad (4)$$

formula yordamida hisoblanadi. Agar $F(x)$ kuch ta'sirida prujinaning siqilish kattaligini x orqali belgilasak, u holda Guk qonuniga ko'ra $F(x)=k\cdot x$ bo'ladi, bu yerda k – o'zgarmas son. (4) formulaga muvofiq bajarilgan ish $A=\int_0^l kxdx=k\cdot\frac{x^2}{2}\Big|_0^l=\frac{kl^2}{2}$. ▲

Xususan, prujinani 0,01 m siqish uchun 10N kuch kerak bo'lsa, $F=10N=k\cdot x$ tenglikdan $k=\frac{F}{x}=1000$. Demak, $F(x)=kx=1000\cdot x$.

Prujinani 0,09 m siqish uchun ketadigan F kuch bajargan ish bu holda

$$A=\int_0^{0.09}1000xdx=1000\cdot\frac{x^2}{2}\Big|_0^{0,09}=500\cdot 0,0081=4,05(J).$$

Mashqlar

51. $y=-x^2+4x$ parabola, (4;0) va (0;4) nuqtalar orqali o'tuvchi to'g'ri chiziq bilan chegaralangan shakl yuzini toping.

52. $f(x)=2x-2$ funksiya grafigi va uning $F(0)=1$ shartni qanoatlantiruvchi boshlang'ich funksiyasi bilan chegaralangan shakl yuzini toping.

Quyidagi chiziqlar bilan chegaralangan shakl yuzini toping. Mos rasm chizing **(53–54)**:

53. 1) $y=x^2,\ y=1-x^2$; 2) $y=\frac{1}{x},\ y=0,\ x=1,\ x=4$;

3) $y=x^2-2x,\ y=4-x^2$; 4) $y=\frac{1}{x},\ y=0,\ x=1,\ x=a(a>1)$.

54. 1) $y=\frac{x^2}{3},\ y=4-\frac{2}{3}x^2$; 2) $y=x^2,\ y=2x^2\ y=2$;

3) $y=x^2,\ y=\frac{x^2}{2},\ y=2x$; 4) $y=\frac{1}{x},\ y=x^2,\ y=\frac{x^2}{2}$.

55. $y=\sin x,\ x\in[0;\pi]$, funksiya grafigining Ox o'qi atrofida aylanishidan hosil bo'lgan jism hajmini hisoblang.

56. $y=\sqrt{x},\ x=1,\ x=4$ chiziqlar bilan chegaralangan shaklning Ox o'qi atrofida aylanishidan hosil bo'ladigan jism hajmini toping.

57. Qiyalik bo'yicha pastga tushayotgan poyezdning tezligi $v(t)=15+0{,}2$(m/s) qonunga ko'ra o'zgaradi. Agar poyezd qiyalikni 20 s davomida o'tgan bo'lsa, qiyalikning uzunligini toping.

58. Vaqtning $t=0$ paytida 20 m/s tezlik bilan yer sirtidan otilgan jism $s(t)=20t-5t^2$ (m) qonun bilan harakatlanadi. Jismning tezligi 5 m/s bo'lganda, u yerdan qanday balandlikda bo'ladi?

59. Avtomobilning tormozlanish tezligi $v(t)=19-1{,}2\cdot t$ (m/s) qonunga ko‘ra o‘zgaradi. Agar avtomobil tormoz olgan vaqtidan 10 s o‘tgach to‘xtagan bo‘lsa, uning tormozlanish yo‘li uzunligini toping.

60. Nuqtaning tezligi $v(t)=3t+\frac{3}{2}\sqrt{t}$ (m/s) qonun bo‘yicha o‘zgaradi. Shu nuqtaning $t=0$ dan $t=4$ gacha vaqt oralig‘ida bosib o‘tgan yo‘lini toping.

61. Yuqoridan $y=e^x$ chiziq bilan, pastdan Ox o‘qi bilan, chapdan $x=0$, o‘ngdan $x=1$ chiziq bilan chegaralangan sohaning Oy o‘qi atrofida aylanishidan hosil bo‘lgan jism hajmini toping.

Quyidagi chiziqlar bilan chegaralangan shakl yuzini toping. Mos rasm chizing **(62–63)**:

62. 1) $y=2\sqrt{x},\ y=6,\ x=0;$ 2) $y=x^2,\ y=2\sqrt{2x};$

3) $y=x^2,\ y=\sqrt[3]{x};$ 4) $y=\sqrt{x},\ y=\sqrt{4-3x},\ y=0.$

63. 1) $y=\sin 6x,\ x=0,\ x=\pi$, Ox o‘qi;

2) $y=\sin 2x,\ x=0,\ x=\frac{\pi}{2}$, Ox o‘qi;

3) $y=\cos x,\ y=1+\frac{2}{\pi}x,\ x=\frac{\pi}{2};$

4) $y=-x^2,\ y=2e^x,\ x=0,\ x=1.$

64*. $y=2x^2-8x$, parabola va shu parabolaga uning uchidan o‘tkazilgan urinma va Oy o‘qi bilan chegaralangan shakl yuzini toping.

65*. $y=x^2+10$ parabola va shu parabolaga (0; 1) nuqtadan o‘tkazilgan urinmalar bilan chegaralangan shakl yuzini toping.

66. Agar $2N$ kuch prujinani 1 cm qissa, prujinani 3 cm qisish uchun sarflanadigan ishni hisoblang.

67. To‘g‘ri chiziqli harakat qilayotgan nuqtaning vaqtning $[t_1;\ t_2]$ oralig‘idagi tezligi $v(t)>0$ bo‘lsin. Vaqtning $t=t_1$ paytidan $t=t_2$ paytigacha bo‘lgan oralig‘ida nuqta bosib o‘tgan yo‘lni toping.

68*. $y=-x^2+1$, $0\le x\le 1$ va Oy o‘qi bilan chegaralangan shaklning Oy o‘qi atrofida aylanishidan hosil bo‘ladigan jism hajmini hisoblang.

69. $y=-x^2+4$, $0\le x\le 2$, $x=0$ (Oy o‘qi) chiziqlar bilan chegaralangan shaklning Ox o‘qi atrofida aylanishidan hosil bo‘lgan jismning hajmini hisoblang.

TAQRIBIY INTEGRALLASH

$\int_a^b f(x)dx$ integraldagi $f(x)$ funksiyaning boshlang'ich funksiyasini topa olsak, uni Nyuton–Leybnis formulasidan foydalanib, aniq hisoblay olamiz.

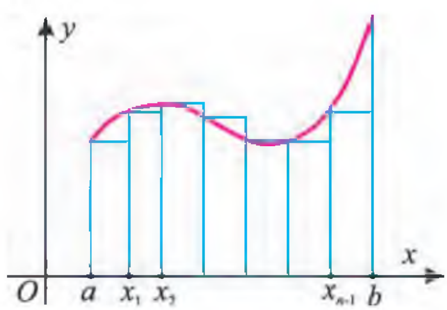
Agar boshlang'ich funksiya topilmasa, u holda $\int_a^b f(x)dx$ integralni taqribiy hisoblash masalasi qo'yiladi. Aniq integralni taqribiy hisoblashning bir nechta usuli bor. Shulardan ba'zilarini keltiramiz.

To'g'ri to'rtburchaklar formulasi. $[a; b]$ kesmada $y=f(x)$ uzluksiz funksiya aniqlangan bo'lsin. $[a; b]$ kesmani $x_0, x_1, x_2, ..., x_{n-1}, x_n$ nuqtalar yordamida n ta o'zaro teng kesmalarga ajratamiz. Har bir kesmaning uzunligi $\Delta x = \frac{b-a}{n}$ ga teng bo'ladi. $a=x_0$, $b=x_n$ deylik. Bo'linish nuqtalari $x_0, x_1, ..., x_{n-1}, x_n$ orqali $y=f(x)$ funksiya grafigi bilan kesishguncha vertikal to'g'ri chiziqlar (Ox ga perpendikularlar) o'tkazamiz. Natijada egri chiziqli trapetsiya n ta kichik egri chiziqli trapetsiyalarga bo'linadi.

Har bir kichik egri chiziqli trapetsiyani asosi Δx, balandligi esa $y=f(x)$ funksiyaning $[x_k; x_{k+1}]$ kesmaning, masalan, chap uchi x_k dagi qiymati $f(x_k)$ ga teng bo'lgan to'g'ri to'rtburchak bilan almashtiramiz, bunda $k=0,1, ..., n-1$.

Hosil bo'lgan bu to'g'ri to'rtburchaklar yuzlarining yig'indisi taqriban egri chiziqli trapetsiyaning yuziga teng bo'ladi (19-rasm). Shunday qilib ushbu formulaga kelamiz:

$$\int_a^b f(x)dx \approx \frac{b-a}{n} \cdot (f(a) + f(x_1) + f(x_2) + ... + f(x_{n-1})). \quad (1)$$



19-rasm.

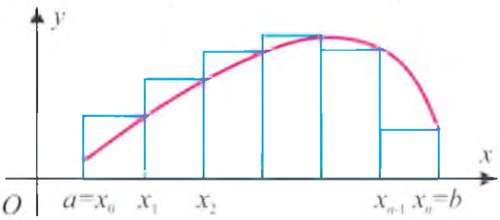
Bu formula aniq integralni taqribiy hisoblashning *to'g'ri to'rtburchaklar formulasi* deyiladi.

To'g'ri to'rtburchakning balandligi sifatida $f(x)$ funksiyaning $[x_k; x_{k+1}]$ kesmaning o'ng uchidagi $f(x_{k+1})$ yoki shu kesma o'rtasi $\frac{x_k + x_{k+1}}{2} = x_{k/2}$ dagi $f(x_{k/2})$ qiymatini ham olish mumkin edi.

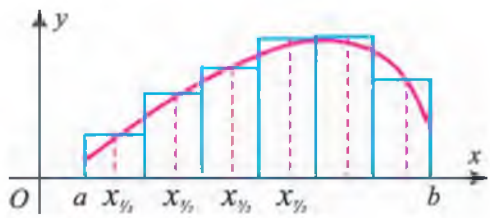
Agar to'g'ri to'rtburchakning balandligi qilib $f(x_{k+1})$ yoki $f(x_{k/2})$ olinsa, u holda, mos ravishda, shunday formulalarni hosil qilamiz (20 a, b -rasmlar):

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\cdot\ (f(x_1)+f(x_2)+\ldots+f(b)), \qquad (1a)$$

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left(f(x_{1/2})+f(x_{3/2})+\ldots+f\left(x_{\frac{2n-1}{2}}\right)\right). \qquad (1b)$$



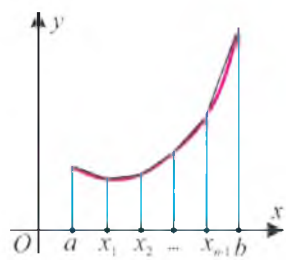
20.a-rasm.



20.b-rasm.

O'tkazilgan vertikal chiziqlarning $y=f(x)$ funksiya grafigi bilan kesishish nuqtalarini ketma-ket tutashtirish natijasida har bir kichik egri chiziqli trapetsiyaning asoslari $f(x_k)$ va $f(x_{k+1})$ hamda balandligi $\Delta x = \frac{b-a}{n}$ bo'lgan trapetsiya bilan almashtiramiz, bunda $k=0,1, \ldots, n-1$.

Hosil qilingan bunday trapetsiyalar yuzlarining yig'indisi taqriban egri chiziqli trapetsiyaning yuziga teng bo'ladi (20.d - rasm).



20.d-rasm.

Shunday qilib ushbu formulani hosil qilamiz:

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\cdot\left(\frac{f(a)+f(b)}{2}+f(x_1)+f(x_2)+\ldots+f(x_{n-1})\right). \qquad (2)$$

Bu formula aniq integralni taqribiy hisoblashning *trapetsiyalar formulasi* deyiladi.

1-misol. $A=\int_0^1 e^{-x^2}dx$ integralni taqribiy hisoblang.

Bizni hizmatdan foydalanib qulay imkoniyatga ega bo'ling!

Bizda maktablar uchun quydagi hujjatlar mavjud

1. **1-11-Sinflar uchun sinf soati ish reja va konspektlari**
2. **1-11-Sinflar uchun barcha fanlardan to'garak hujjatlari**
3. **Sinf rahbar hujjatlari**
4. **Metodbirlashma hujjatlari**
5. **Ustama hujjatlari**
6. **1-11-Sinflar uchun barcha fanlardan konspektlar**
7. **1-11-Sinflar uchun Ish rejalar (Taqvim mavzu rejalar)**
8. **Darsliklarning elektron varianti**
9. **Maktab ish hujjatlari**
10. **Direktor ish hujjatlari**
11. **MMIBDO' ish hujjatlari**
12. **O'IBDO' ish hujjatlari**
13. **Psixolog hujjatlari**
14. **Xotin-qizlar qo'mitasi ish hujjatlari**
15. **Kutubxona mudirasi ish hujjatlari**
16. **Besh tashabbus hujjatlari**
17. **Ochiq dars ishlanmalar, taqdimotlar, slaydlar**